Formulas you will be given – know what they mean and how to use them

- For binomial random variable: m = np and s = $\sqrt{npq}$
- For a discrete distribution: $\mu = \Sigma[xP(x)]$
- $z = \dfrac{observation - \mu}{\sigma}$

1) Consider the following discrete distribution for the number of books read over the summer for Stat 201 students.

Number of Books	P(x)
0	.5
1	.1
2	.1
3	.1
4	.1
5	.1

a. Show that this is a valid discrete distribution.

① X, the number of books is discrete

② All P(x) are between ∅ and 1 $(0 \le P(x) \le 1)$

③ $\Sigma P(x) = .5 + .1 + .1 + .1 + .1 + .1 = 1$

b. Find the mean of this distribution.

$$\mu = \Sigma x P(x) = (0 * .5) + (1 * .1) + (2 * .1) + (3 * .1) + (4 * .1) + (5 * .1)$$
$$= 0 + .1 + .2 + .3 + .4 + .5$$
$$= 1.5$$

2) Consider the following discrete distribution for the an unfair die at a casino

Die side	P(x)
1	.025
2	.4
3	.4
4	.1
5	.025
6	.05

a. Show that this is a valid discrete distribution.

① The die side variable is discrete

② All P(x) are between 0 and 1 $(0 \le P(x) \le 1)$

③ $\sum P(x) = .025 + .4 + .4 + .1 + .025 + .05 = 1$

b. Find the mean of this distribution. Why might this unfair die be good for the casino?

$$\mu = \sum x P(x) = (1 * .025) + (2 * .4) + (3 * .4) + (4 * .1) + (5 * .025) + (6 * .05)$$

$$= .025 + .8 + 1.2 + .4 + .125 + .3$$

$$= 2.85$$

This might be good for the casino because it has a lower mean than a regular die, 3.5. We also note that 1, 5 and 6 have low probability which might help favor the casino in some games.

3) The number of ounces of coffee drank per week by college students follows a symmetric, bell shaped curve with a mean of 60oz and a standard deviation of 25oz.

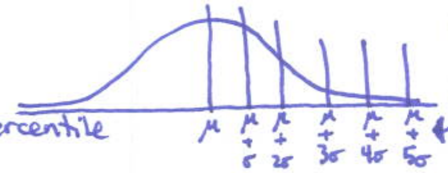
a. According to the empirical rule, 84% of the data will lie below what value?

84% of the data lies below $\mu + 1\sigma = 60 + 25 = 85$ oz.

b. I drink about 185oz of coffee per week, what percentile would I be in?

$$Z_{185} = \frac{185-60}{25} = \frac{125}{25} = 5$$

$P(z<5) \approx 1 = 100\% \rightarrow$ The 100th percentile



Almost all of the data is below 185oz

c. How much coffee per week does the person at the 10th percentile drink?

① $Z_{10\%} = -1.28$ (This gives us 10.03% but it's the closest we can get)

② $-1.28 = \frac{x-60}{25} \rightarrow x-60 = 25(-1.28) \rightarrow x = 25(-1.28)+60 = 28$

ANS 28 ounces of coffee per week puts someone in the 10th percentile.

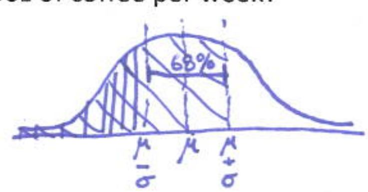
d. What percentage of students drink between 35oz and 85oz of coffee per week?

① $Z_{35} = \frac{35-60}{25} = \frac{-25}{25} = -1 \rightarrow .1587$ ⁝ $.8413$

② $Z_{85} = \frac{85-60}{25} = \frac{25}{25} = 1 \rightarrow .8413$ ⁝ $-.1587$

Puzzle pieces

↑ Percentiles $.6826 \rightarrow \boxed{68.26\%}$



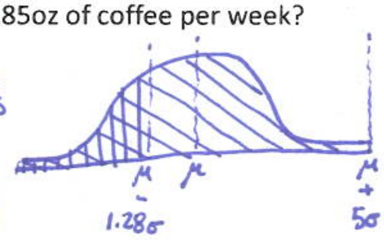
e. What percentage of students drink between 28oz and 185oz of coffee per week?

① $Z_{28} = \frac{28-60}{25} = \frac{-32}{25} = -1.28 \rightarrow .1003$ ⁝ 1.00

② $Z_{185} = \frac{185-60}{25} = \frac{125}{25} = 5 \rightarrow 1.00$ ⁝ $-.1003$

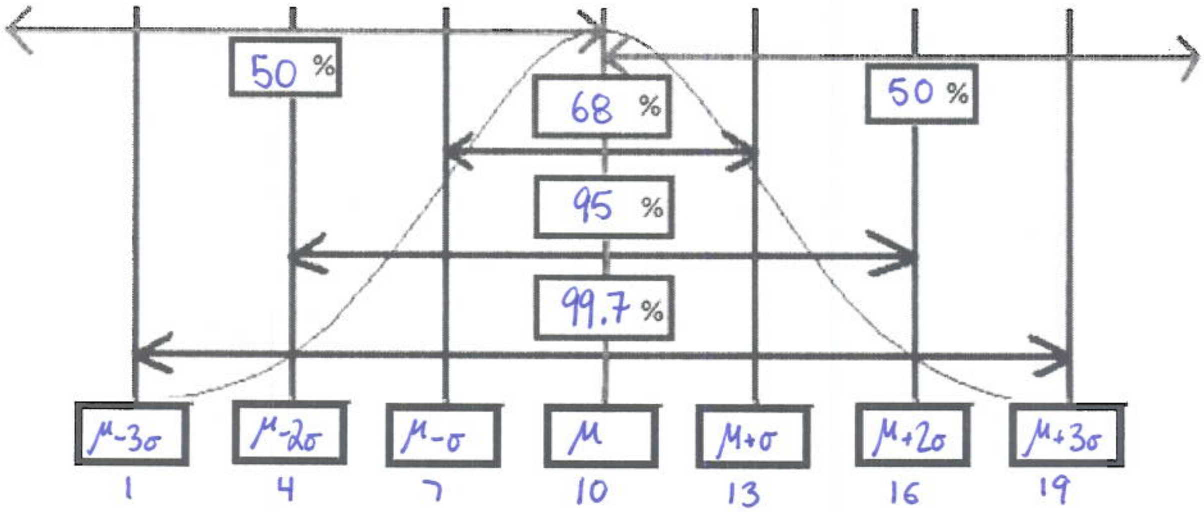
Puzzle pieces

↑ Percentiles $.8997 \rightarrow \boxed{89.97\%}$



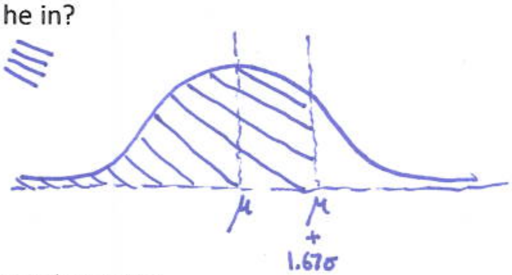
4) The number of goals scored, per season, by a EPL soccer player is 10 with a standard deviation of 3.

a. Do out all that empirical rule stuff.



b. If a player scored 15 goals, what percentile is he in?

$Z_{15} = \frac{15-10}{3} = \frac{5}{3} = 1.\overline{66} \approx 1.67 \longrightarrow .9525$



c. How many goals did someone at the 25th percentile score?

① $Z_{25\%} = -.67$ (This gives us 25.14% but it's the closest we can get)

② $-.67 = \frac{x-10}{3} \longrightarrow x-10 = 3(-.67) \longrightarrow x = 3(-.67)+10 = 7.99$

ANS About 8 goals puts a player at the 25th percentile

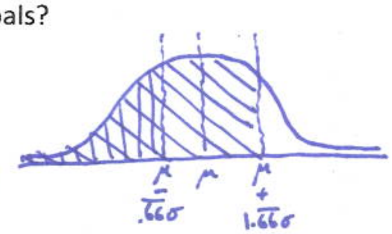
d. What percentage of players scored between 8 and 15 goals?

① $Z_8 = \frac{8-10}{3} = \frac{-2}{3} = -.\overline{66} \approx -.67 \longrightarrow .2514$

② $Z_{15} = \frac{15-10}{3} = \frac{5}{3} = 1.\overline{66} \approx 1.67 \longrightarrow .9525$

↑ percentiles

$.9525 - .2514 = .7011 \rightarrow \boxed{70.11\%}$



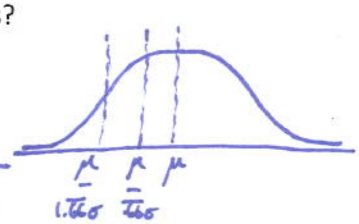
e. What percentage of players scored between 5 and 8 goals?

① $Z_5 = \frac{5-10}{3} = \frac{-5}{3} = -1.\overline{66} \approx -1.67 \rightarrow .0475$

② $Z_8 = \frac{8-10}{3} = \frac{-2}{3} = -.\overline{66} \approx -.67 \longrightarrow .2514$

↑ percentiles

$.2514 - .0475 = .2039 \rightarrow \boxed{20.39\%}$



5) Assuming no one misses the exam, there should be forty eight of you taking the test. From past experience I expect about 90% of you to pass. Let X= the number of you that pass.

a. Find the mean of x, the average number of students that will pass.

$n=48$

$p=.9$ $\qquad\qquad \mu = np = 48 \cdot .9 = 43.2$

$q=1-p=.1$

b. Find the standard deviation of x.

$n=48$

$p=.9$ $\qquad\qquad \sigma = \sqrt{npq} = \sqrt{48(.9)(.1)} = \sqrt{4.32} = 2.0785$

$q=1-p=.1$

c. What is the probability that exactly forty seven people pass?

$$P(x=47) = \frac{48!}{47!(48-47)!} \cdot .9^{47} \cdot .1^{48-47} = \frac{(48 \cdot \cancel{47} \cdot \cancel{46} \cdots \cancel{1})}{(\cancel{47} \cdot \cancel{46} \cdot \cancel{45} \cdots \cancel{1})(1)} \cdot .9^{47} \cdot .1^{1}$$

$$= \frac{48}{1} \cdot .9^{47} \cdot .1^{1} = 48(.00706965)(.1) = .0339343$$

d. What's the probability that all forty eight students pass?

$$P(x=48) = \frac{48!}{48!(48-48)!} \cdot .9^{48} \cdot .1^{48-48} = \frac{\cancel{48} \cdot \cancel{47} \cdot \cancel{46} \cdots \cancel{1}}{(\cancel{48} \cdot \cancel{47} \cdot \cancel{46} \cdots \cancel{1})(1)} \cdot .9^{48} \cdot .1^{0}$$

$$= .9^{48}(1) = .0063627$$

e. What's the probability more than forty six students pass?

$$P(x>46) = P(x=47) + P(x=48) = .0339343 + .0063627 = .040297$$

6) According to teausa.com 65% of the tea brewed in the US were brewed using teabags.

a. What's the probability that a randomly selected tea was brewed using something other than a tea bag?

$1 - .65 = .35 \rightarrow$ 35% of the teas were brewed using something other than a tea bag

b. Say we take a random sample of ten teas, what's the probability that all ten of them are brewed using something other than a tea bag?

$n = 10$
$p = .35$
$q = 1 - p = .65$

$$P(x=10) = \frac{10!}{10!(10-10)!} .35^{10} .65^{10-10} = \frac{\cancel{10 \cdot 9 \cdot 8 \cdots 1}}{\cancel{10 \cdot 9 \cdot 8 \cdots 1}\,(1)} .35^{10} .65^{0}$$

$$= .35^{10}(1) = \boxed{.000027585}$$

c. Say we take a random sample of ten teas, what's the probability that all ten of them are brewed using a tea bag?

$n = 10$
$p = .35$
$q = 1 - p = .65$

$$P(x=0) = \frac{10!}{0!(10-0)!} .35^{0} .65^{10-0} = \frac{\cancel{10 \cdot 9 \cdot 8 \cdots 1}}{(1)\cancel{(10 \cdot 9 \cdot 8 \cdots 1)}} (1) .65^{10}$$

$$= .65^{10} = \boxed{.0134627}$$

d. Say we take a random sample of ten teas, what's the probability that five of them are brewed using something other than a tea bag?

$n = 10$
$p = .35$
$q = 1 - p = .65$

$$P(x=5) = \frac{10!}{5!(10-5)!} .35^{5} .65^{10-5} = \frac{10 \cdot 9 \cdot 8 \cdots 1}{(\cancel{5 \cdot 4 \cdot 3 \cdots 1})(5 \cdot 4 \cdot 3 \cdots 1)} .35^{5} .65^{5}$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} .35^{5} .65^{5} = \frac{41160}{120} .35^{5} .65^{5} = 343(.0052521875)(.1160290625)$$

$$= \boxed{.209026}$$

e. Say we take a random sample of ten teas, what's the probability that more than five of them are brewed using something other than a tea bag?

$$P(x > 5) = P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)$$

$$= .01622 + .00309 + .0003862 + .00002861 + .000027585$$

$$\approx .01973$$

7) From past experience, it rains about 10% of the days in Columbia, SC. Let X=# of sunny days and consider looking at how many days will be sunny in the next two years, 730 days.

f. What kind of problem is this? What parameters are defined and what are they defined as?

This is a binomial problem with: $n = 730$

$p = .9$

$q = .1$

g. Find the mean of x, what does it mean?

$\mu = np = 730(.9) = 657$ This is the average/expected value of sunny days in the 730 days sampled.

h. Find the standard deviation of x.

$\sigma = \sqrt{npq} = \sqrt{730(.9)(.1)} = \sqrt{65.7} = 8.10555365$

8) Eighty percent of college students have tried an illegal drug: marijuana, prescription drugs, etc. Consider taking a random sample of 100 college students. Can we use the binomial experiment here? If so, find the mean and standard deviation of this distribution.

Yes we can: · Fixed $n = 100$

· Identical trials $\rightarrow p = .8$, $q = 1 - p = .2$

· Independent trials

$$\mu = np = 100(.8) = 80$$

$$\sigma = \sqrt{npq} = \sqrt{100(.8)(.2)} = \sqrt{16} = 4$$

9) Show that the empirical rule works using the z table.

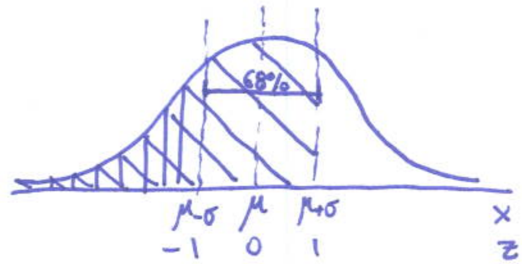
Percentile

$Z = 1 \longrightarrow .8413$

$Z = -1 \longrightarrow \underline{-.1587}$

$.6826 \longrightarrow 68.26\%$

$.26\%$ Error

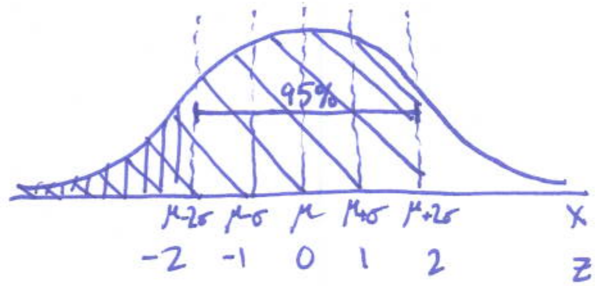


$Z = 2 \longrightarrow .9772$

$Z = -2 \longrightarrow \underline{-.0228}$

$.9544 \longrightarrow 95.44\%$

$.44\%$ Error



$Z = 3 \longrightarrow .9987$

$Z = -3 \longrightarrow \underline{-.0013}$

$.9974 \longrightarrow 99.74\%$

$.04\%$ Error